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Market selection decisions for inventory models with price-sensitive demand

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Abstract In the majority of classical inventory theory literature, demand arises from exogenous sources upon which the firm has little or no control. In many practical contexts, however, aggregate demand is comprised of individual demands from a number of distinct customers or markets. This introduces new dimensions to supply chain planning problems involving the selection of markets or customers to include in the demand portfolio. We present a nonlinear, combinatorial optimization model to address planning decisions in both deterministic and stochastic settings, where a firm constructs a demand portfolio from a set of potential markets having price-sensitive demands. We first consider a pricing strategy that dictates a single price throughout all markets and provide an efficient algorithm for maximizing total profit. We also analyze the model under a market-specific pricing policy and describe its optimal solution. An extensive computational study characterizes the effects of key system parameters on the optimal value of expected profit, and provides some interesting insights on how a given market's characteristics can affect optimal pricing decisions in other markets.

Keywords Economic order quantity · Newsvendor problem · Market selection · Pricing

1 Introduction

Standard approaches to classical inventory control problems typically assume that the firm has no effect on the revenue and demand parameters. Although this may be reasonable in

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perfectly competitive environments where firms are price-takers, in many contexts firms can manipulate demand to a certain degree using marketing tools such as pricing. In such contexts, a supplier of a good must often determine the good's price in addition to inventory policy parameters in order to respond to the implied demand. Another limitation of past standard models is that the firm effectively serves a single market. Although a few studies (discussed in the following literature review section) exist that address multiple markets or locations, these also assume that the firm does not have the ability to choose whether or not to serve a market.

A considerable body of literature proposes extensions addressing the limitations of these models. The vast majority of these studies assume either exogenous demand and revenue, or a predetermined market portfolio. In this study, we introduce an optimization model that relaxes these assumptions. Our model applies to and generalizes related studies in the literature both in deterministic (Economic Order Quantity [EOQ] model) and the stochastic settings (Newsvendor model).

We consider a profit-maximizing firm offering a single product. A set of potential markets exists, and the firm must decide whether or not to serve each market (although we consider distinct markets, the setting also applies to a single-market problem with different customer classes). Revenue in each market (or expected revenue, in the stochastic case) is a function of the price offered. The firm must determine the markets it will serve, the price (or the prices in each market), and a procurement quantity that will be used to supply the selected markets. The resulting profit maximization problem is quite different from standard inventory control problems. In the traditional settings, the optimal inventory control policy parameter value(s) depends on a predefined demand rate (deterministic setting), or an exogenous probability distribution of demand (stochastic setting), over which the firm has no control. In the contexts we consider, however, the demand rate (or probability distribution) depends on the markets the firm selects and the selling price (or prices) offered in these markets. Moreover, since stock is pooled for all selected markets in the stochastic case, and fixed costs are shared in the deterministic case, the profitability of serving a market depends on the entire set of markets selected. As a result, in addition to containing nonlinear operations (EOQ/newsvendor) and economics (pricing) elements, the model we develop also contains a substantial combinatorial optimization component.

In the absence of explicit constraints, a profit maximizing firm would naturally prefer to set different prices in different markets, assuming the market characteristics are different. However, the associated marketing and operational costs under market-specific prices may be higher than those in the single-price case. Even if such costs are not an issue, a firm might choose to set the same price in all markets as a marketing strategy in order to maintain a certain brand reputation and/or consistent customer experience. A number of additional reasons may also prevent a firm from applying a 'market-specific pricing' strategy. For instance, if the supplier uses regional distributors to supply different markets, in order to avoid conflicts with distributors and ensure equity, the supplier may apply a single price for all distributors (see, e.g., Balakrishnan et al. [3]). Alternatively, regulations may exist that prevent charging different prices for the same good in different markets or to different customer classes (see, e.g., Cabral [5] for a uniform price imposed by an antitrust authority).

This paper therefore studies an optimization problem that applies to both deterministic and stochastic inventory problems, and analyzes simultaneous market selection, pricing, and order quantity decisions for two potential cases: (i) the firm must offer the same selling price in all markets selected, and (ii) the firm has the flexibility to offer market-specific prices. At first glance, this study seems to be closely related to a branch of economics literature on multiproduct monopoly pricing and third degree price discrimination (see, e.g., Tirole [32] for a brief description of these problems). As we will discuss in more detail in Sect. 2, our model not only differs from this literature by modeling operational costs in inventory systems, but it incorporates explicit market selection decisions and provides an efficient algorithm for solving multi-market problems as well, which has not been done in the economics literature to our knowledge.

Under mild assumptions on the revenue and cost functions, we provide a polynomial-time solution for the single-price strategy and characterize the optimal solution for the market-specific pricing strategy. (In particular, we assume that a market's revenue depends only on that market's price. This provides some limits on the model's ability to account for broader economic effects. That is, the revenue functions do not explicitly account for externalities such as income and substitution effects. First, we assume that markets are sufficiently geographically dispersed such that prices in other markets do not impact a given market's revenue. Moreover, assuming that a market's revenue depends only on the market's price implies that income effects are fixed, for example, in the short run, no substitute exists for the product, and/or that the revenue functions implicitly account for income and substitution effects in equilibrium.) These models can be applied as benchmarks for making market selection, pricing, and procurement quantity decisions in stochastic environments with a short selling season, and deterministic environments with continuous and stationary demand. Using these models, we perform an extensive computational analysis to demonstrate the effects that different critical parameter settings have on the optimal value of (expected) profit. The results of this analysis provide some interesting and, in some cases, unexpected insights on how a market's characteristics can affect pricing decisions in other markets.

The remainder of this paper is organized as follows. In Sect. 2, we provide a brief review of studies on pricing and market/order selection decisions in the operations literature. We introduce a general problem framework and key modeling assumptions in Sect. 3. Section 4 proposes solution approaches for the 'single-price' and 'market-specific pricing' strategies, respectively. We provide an extensive computational study and present our main findings in Sect. 5. In Sect. 6, we summarize our work and suggest further research directions.

2 Literature review

The EOQ and Newsvendor models are regarded as the building blocks of deterministic and stochastic inventory theory, respectively. Despite their restrictive assumptions, these models have been extensively utilized in practice due to their simple structures and robust performance. An important extension to such classical inventory control problems permits a price-dependent demand process. The first model of this kind was formulated by Whitin [35], who incorporated pricing into operations management under both EOQ and Newsvendor settings. The EOQ version was later explicitly solved by Porteus [25]. Abad [1] addresses a similar problem for a more general demand function with perishable items and partial backordering. Lau and Lau [17] investigate a joint-pricing model in the absence of setup costs. Viswanathan and Wang [34] model a single-retailer, single-supplier channel, where the retailer faces price-sensitive demand. Ray et al. [26] consider two pricing approaches (price as a decision variable and mark-up pricing) and concentrate on identifying managerial insights regarding the behavior of the optimal decisions.

Whitin [35] appears to be the first study that links the newsvendor problem with pricing decisions. Mills [20] also assumes that demand is a random variable with an expected value that is decreasing in price, where randomness is modeled in an additive fashion with a constant variance. Lau and Lau [16] consider two different approaches to model demand randomness:

(i) a simple homoscedastic regression model, $d(p) = a - bp + X$, and (ii) a demand distribution that is constructed using a combination of statistical data analysis and experts' subjective estimates. Demand randomness is usually modeled either in an additive ($D(p) = q(p) + X$) or a multiplicative ($D(p) = q(p)X$) fashion, where $q(p)$ is a nonincreasing function of price and X is a random factor. Petruzzi and Dada [21] analyze both cases in detail and highlight the structural differences between these models. Young [36] proposes a demand model that handles both cases, and investigates the behavior of the optimal decision with respect to uncertainty. Polatoglu [24] emphasizes the limitations of both the additive and multiplicative models, and formulates the problem with a general demand distribution to characterize the properties of the model, independent of the way randomness is handled.

Note that all of these models consider a single market, i.e., the firm faces a single stream of demand. Eppen [12] provides an early study considering a multi-location newsvendor problem where demands are normally distributed. Chen and Lin [8] introduce general distributions for the demands and concave holding and shortage costs. Chang and Lin [7] extend this work by considering transportation costs. Cherikh [11] and Lin et al. [19] employ a profit maximization perspective. Chen et al. [9,10] appear to be among the earliest studies that consider pricing and inventory control in a multilocation setting. They assume that demands occur at a constant deterministic rate that depends on the price and consider 'location specific pricing'. Federgruen and Heching [13] consider a periodic review, stochastic model with multiple retailers where demand at a given retailer is price-sensitive. They consider a single-price strategy in a given period and develop an approximate model that is tractable. None of these models, however, consider market selection together with pricing, and they thus assume that the firm must satisfy all markets (or retailers).

In market selection problems, on the other hand, the firm has the flexibility to select the demands it will serve. Geunes et al. [14] generalize the classical EOQ and EPQ models to address economic ordering decisions when a producer can choose whether to satisfy multiple markets. Geunes et al. [15] consider order selection in a multi-period lot-sizing context with pricing decisions. They also incorporate limited capacity in a requirements planning model with order selection or pricing. Carr and Lovejoy [6] introduce the 'inverse newsvendor problem' where the firm chooses an optimal demand portfolio with a fixed but uncertain capacity. The only study in the literature that considers market selection with demands dependent on endogenous variables is introduced by Taaffe et al. [31]. They present the 'selective newsvendor problem' (SNP), which addresses integrated market selection and ordering decisions where demand in each market is normally distributed and dependent on the marketing effort exerted. However, they only consider special functional forms of the relationship between the demand and marketing effort. These functional forms allow the market selection and marketing effort decisions to be separable. In our case, on the other hand, we allow general forms of revenue and cost functions, and market selection and pricing decisions are not separable. Furthermore, Taaffe et al. [31] do not consider a setting where the endogenous variable is restricted to take the same value in all markets, which would be of little value in the marketing effort context.

The fundamental difference between our model and existing market selection models is that we introduce a general optimization model and an efficient solution approach that not only incorporate endogenous pricing decisions but also apply to both deterministic and stochastic settings under certain assumptions. Hence, the firm must determine an optimal price (in each market or a single price for all markets), together with the market selection and inventory control policy parameter decisions. Based on the characteristics of the markets and the supplier's policies, the firm may choose to set a single price throughout all selected markets or an individual price for each selected market. To our knowledge, this is the first study that analyzes single-price and market-specific pricing schemes in a market selection context involving operations-related costs.

A stream of economics literature exists that deals with pricing decisions for a multi-market monopoly, which is referred to as 'third degree discrimination'. Third degree price discrimination can be broadly defined as 'charging different consumers (markets) different prices for the same good' (Armstrong [2]), and has been studied since the 1920s (see Pigou [23] and Robinson [27] for the earliest discussions). This stream of literature also investigates the differences between third degree price discrimination, which corresponds to our 'market-specific pricing strategy', and 'uniform pricing', which corresponds to our 'single-price strategy', in terms of output and welfare implications (see, e.g., Battalio and Ekelund [4], Schmalansee [29] and Varian [33]). The reader may refer to Phlips [22] and Armstrong [2] for detailed reviews of studies on price discrimination. Our study differs from this stream in important ways. First of all, we consider general forms of revenue and cost functions, that in certain settings correspond to different supply chain management problems. The explicit consideration of operations costs has not been considered in this stream of the economics literature. We also model market selection decisions explicitly. Although there are studies in the economics literature that recognize the fact that single-price strategy may exclude some of the markets (see, e.g., Battalio and Ekelund [4] and Layson [18]), these studies consider a setting with only two markets and assume linear costs. Furthermore, the price variable dictates the market selection decisions in these studies. That is, a market is considered not served only if its demand at the optimal price level is zero. Our study, on the other hand, not only recognizes this phenomenon, but allows the firm to not serve a market that would have positive demand at the uniform price level offered in other markets. Furthermore, we provide an algorithm to solve this more general problem efficiently, which is not addressed in the economics literature.

3 Problem description and assumptions

In this section, we formally state our assumptions, and describe and formulate the model. Let n denote the number of potential markets available for a supplier to serve. The total (expected) revenue from market i is price dependent and is denoted by a continuous function, $R_i(p_i)$, where p_i is the price in market i. Let y denote the market selection vector, i.e., $y_i = 1$ if the firm decides to serve market i, and $y_i = 0$ otherwise. The total (expected) cost the supplier incurs when serving market i in isolation using a unit price p_i equals $S \times C_i(p_i)$, where S is a cost parameter that is context dependent, and $C_i(p_i)$ is a continuous, decreasing function of p_i. Moreover, let $\bar{p}$ denote the vector of n market prices and denote the total cost incurred for serving all selected markets by $C(\bar{p}, y)$. We will consider settings in which the individual market costs are not independent of one another when multiple markets are selected, so that $C(\bar{p}, y) \neq \sum_{i=1}^{n} C_i(p_i) y_i$. Instead, we assume that the total cost function is of the form

$$C(\bar{p}, y) = S \sqrt{\sum_{i=1}^{n} [C_i(p_i)]^2 y_i}. \tag{1}$$

This cost structure encompasses both risk pooling in stochastic settings (such as the newsvendor context discussed in Sect. 3.1) and economies-of-scale in procurement costs in deterministic settings (such as the EOQ context discussed in Sect. 3.2). The market selection problem with pricing (MSP) can then be constructed as follows:

$$\begin{aligned} \max \quad & G(\bar{p}, y) = \sum_{i=1}^{n} R_i(p_i) y_i - S \sqrt{\sum_{i=1}^{n} [C_i(p_i)]^2 y_i} \\ \text{subject to} \quad & y_i \in \{0, 1\}, \qquad \forall i = 1, \dots, n, \\ & \bar{p} \in P. \end{aligned}$$

The definition of the set P depends on whether we consider the single-price case (in which case P consists of all vectors $\bar{p} \in \mathbb{R}^n$ such that $p_i = p$ for all $i = 1, \dots, n$ and for some $p \in \mathbb{R}$) or the market-specific pricing case (in which case $P = \mathbb{R}^n$). We next discuss how the above formulation applies to different modeling environments.

3.1 Newsvendor model with market selection and pricing

(MSP) applies to a single-period, stochastic inventory problem under certain assumptions. Consider a set of potential markets where demand in market i is random and price-sensitive. Let the distribution of the random factor depend on price. In particular, we model demand in market i as $D_i(p_i) = q_i(p_i) + X_i$. The X_i's are independent random variables having pdf and cdf $f_i(x, p_i)$ and $F_i(x, p_i)$. Note that this demand model is quite similar to Young [36], who formulates the demand as $\alpha(p)\epsilon + \beta(p)$ where $\alpha(p)$ and $\beta(p)$ are deterministic functions of p and ϵ is a random variable. In the Appendix, we demonstrate that the multiplicative and additive models can be equivalently represented by one another in our setting when the X_i's are independent, normally distributed random variables. Hence, we restrict our analysis to the additive randomness case, noting that similar arguments and results are also valid for the multiplicative case. Next, we discuss the assumptions and their implications, under which (MSP) can handle the Newsvendor problem with market selection and pricing.

Assumption 1 There is effectively a single pool of stock that serves all markets.

Assumption 1 states that stock is allocated among selected markets after the uncertainty is resolved. This is quite reasonable when the markets are close to each other or when the firm offers the product on a ship-to-order basis. Inventory pooling naturally follows if individual markets represent different customer segments in a single market. Note that the problem would be trivial if inventory was not pooled, since each market would be considered separately in terms of inventory, pricing and selection decisions.

Assumption 2 The random element in market i, X_i, is normally distributed with mean 0, and standard deviation $\sigma_i(p_i)$.

Although the support of the normal distribution is the entire real line, and hence it may result in negative demand occurrences, it is widely employed in the literature. Furthermore, we can limit the possibility of negative demand to negligible levels since the distribution of the random factor also depends on the price level. Moreover, if each market's demand is comprised of a large number of individual demands from different customers, then we would expect that the distribution of demand in each market can be closely approximated by a normal distribution as a result of the Central Limit Theorem (see, e.g., Ross [28], p. 79). Assuming $E[X_i] = 0$ is not restrictive since any nonzero mean value may be incorporated into the deterministic part of the demand, $q_i(p_i)$, without any effect on the model. The normality assumption enables us to model the aggregate demand explicitly since the sum of independent normal random variables is also normally distributed; that is, the aggregate demand is normally distributed with mean $D_y(\bar{p}) = \sum_{i=1}^{n} q_i(p_i)y_i$ and standard deviation $\sigma_y(\bar{p}) = \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i)y_i}$.

Assumption 3 Shortages are either expedited or backlogged until the end of the selling season by placing an additional order that arrives at end of the selling season. In either case, the cost per shortage is independent of price.

Note that in the backlogging case, Assumption 3 can be interpreted as having customers who are willing to wait until the end of the selling season to receive the product when the supplier faces a shortage.

Assumption 3, together with Assumption 2, results in separate inventory and pricing decisions as follows: recall that, by Assumption 2, the aggregate demand is normally distributed with mean $q_y(\bar{p}) = \sum_{i=1}^{n} q_i(p_i)y_i$ and standard deviation $\sigma_y(\bar{p}) = \sqrt{\sum_{i=1}^{n} \sigma_i^2(p_i)y_i}$. Hence, the inventory decision is equivalent to a classical newsvendor problem. Let $z = (Q - q_y(\bar{p}))/\sigma_y(\bar{p})$, where Q is the procurement quantity from an external supplier. Then, the expected shortages and leftovers are given by $\sigma_y(\bar{p})E[\epsilon - z]^+$ and $\sigma_y(\bar{p})E[z - \epsilon]^+$, respectively, where ϵ is a standard normal random variable. By Assumption 2, the expected sales are directly equal to the mean demand, i.e., $q_y(\bar{p})$. Hence, the expected total profit can be written as

$$G(\bar{p}, y, z) = \sum_{i=1}^{n}(p_i - c)q_i(p_i)y_i - K(z)\sqrt{\sum_{i=1}^{n}\sigma_i^2(p_i)y_i}, \tag{2}$$

where $K(z) = (c - v)E[z - \epsilon]^+ + (e - c)E[\epsilon - z]^+$, and c, e, v are per unit procurement cost, per unit shortage cost and per unit salvage value, respectively. Note that we have y_i in the square root term since $y_i^2 = y_i$ as $y_i \in \{0, 1\}$. Since per unit shortage cost is independent of price, the optimal value of z is independent of the prices set in the selected markets, and hence it is also independent of the market selection decisions; that is $z^* = s(\rho)$, where $s(\rho)$ is the standard normal variate associated with the fractile $\rho = (e - c)/(e - v)$. Therefore, the newsvendor problem with market selection and pricing reduces to

$$\begin{aligned} \max \quad & G(\bar{p}, y) = \sum_{i=1}^{n}(p_i - c)q_i(p_i)y_i - K^*\sqrt{\sum_{i=1}^{n}\sigma_i^2(p_i)y_i} \\ \text{subject to} \quad & y_i \in \{0, 1\}, \qquad \forall i = 1, \ldots, n, \\ & \bar{p} \in P, \end{aligned}$$

where $K^* = K(z^*)$. Note that this problem is a special case of (MSP), where $R_i(p_i) = (p_i - c)q_i(p_i)$, $S = K^*$, and $C_i(p_i) = \sigma_i(p_i)$.

3.2 EOQ with market selection and pricing

Geunes et al. [14] consider a standard EOQ problem with two exceptions. First, the producer can choose whether or not to satisfy each market's demand. Second, instead of minimizing average annual cost, they maximize average annual net profit. The resulting model, called the EOQ model with market choice (EOQMC), is

$$\begin{array}{ll} \max & G(y) = \sum_{i=1}^{n} r_i \lambda_i y_i - \sqrt{2Kh \sum_{i=1}^{n} \lambda_i y_i} \\ \text{subject to} & y_i \in \{0, 1\}, \quad \forall i = 1, \ldots, n, \end{array}$$

where r_i and λ_i denote the per unit net revenue and demand rate in market i, respectively, h denotes inventory holding cost rate, and K denotes the fixed setup/order cost.

Note that (EOQMC) is a special case of (MSP) described earlier, where $R_i(p_i) = r_i\lambda_i$, $S = \sqrt{2K}$, and $C_i(p_i) = \sqrt{\lambda_i h}$. Moreover, (MSP) can easily incorporate a price-sensitive demand rate by setting $R_i(p_i) = (p_i - c_i)\lambda_i(p_i)$ and $C_i(p_i) = \sqrt{\lambda_i(p_i)h}$. Allowing different holding costs for markets can easily be handled by simply replacing h by h_i. As a result, (MSP) is capable of handling (EOQMC) described in Geunes et al. [14], together with a number of its generalizations.

4 Solution algorithms for (MSP)

The previous section demonstrated that (MSP) generalizes the EOQ and newsvendor models with market selection and pricing under certain assumptions. In this section, we seek efficient solution algorithms for (MSP), which is a nonlinear, combinatorial optimization problem. We first consider (MSP) under a single-price strategy, i.e., the firm chooses to offer the product at the same price in all selected markets. This corresponds to the case in which the set P is limited to all vectors in $\mathbb{R}^n$ such that all n elements are identical, and we refer to this problem as (MSP-S). After introducing the algorithm for (MSP-S), we then consider the market-specific pricing strategy, which we refer to as (MSP-MS) from this point onward.

4.1 Market selection with a single price—(MSP-S)

The market selection and pricing problem when the firm is required to set the same price in all markets selected can be formulated using a single price variable p as

$$\begin{array}{ll} \max & G(p, y) = \sum_{i=1}^{n} R_i(p) y_i - S \sqrt{\sum_{i=1}^{n} [C_i(p)]^2 y_i} \\ \text{subject to} & y_i \in \{0, 1\}, \quad \forall i = 1, \ldots, n. \end{array}$$

Given a price, (MSP-S) reduces to a general version of the selective newsvendor problem (SNP), which is discussed by Taaffe et al. [31]. Hence, for a fixed price, we can solve the market selection problem employing the *Decreasing Expected Revenue to Uncertainty Ratio Property* introduced in Taaffe et al. [31] (based on a result from Shen et al. [30]), where the uncertainty in a market is characterized by its variance.

Property 1 (cf. Taaffe et al. [31]) *After indexing markets in nonincreasing order of the ratio of expected net revenue to uncertainty, an optimal solution to [SNP] exists such that if we select market ℓ, we also select markets* $1, 2, \ldots, \ell - 1$.

Following Property 1 and sorting markets in nondecreasing order of the ratio of the (expected) revenue ($R_i(p)$) to the cost contribution ($[C_i(p)]^2$), an optimal solution to the market selection problem with a fixed price level can be found by selecting the best of n candidate solutions, where candidate solution ℓ selects markets 1 to ℓ (see Taaffe et al. [31] for details). Note that the sorting mechanism works in favor of markets that have greater revenue and less cost contribution, which satisfies intuition.

For (MSP-S), however, price is a decision variable and the ordering of markets may differ at different price levels. To overcome this problem, the sorting scheme characterized above can be utilized to divide the feasible region in price into a set of contiguous, non-overlapping intervals, where the preference order of markets does not change within an interval. Hence, within each interval, we can utilize Property 1 with a slight modification to obtain an optimal set of markets for the price interval. In particular, for each candidate solution in each price interval, we need to maximize the objective function with respect to p with the constraint that p falls in the specified interval. The price intervals that enable this approach can be generated as follows.

Let P_{ij} denote the set of critical price levels where the preference ratios for markets i and j are equal, i.e., the threshold prices beyond which the order of these markets is reversed in the sorting scheme. For all (i, j) pairs such that $i < j$, P_{ij} is given by

$$P_{ij} = \left\{ c \leq p \leq \min(p_i^0, p_j^0) : \frac{R_i(p)}{[C_i(p)]^2} = \frac{R_j(p)}{[C_j(p)]^2} \right\}. \tag{3}$$

In the specific applications that we considered in the previous section, demand in market i may be zero beyond a price level, p_i^0. In order to address this issue, we add these p_i^0 values to the critical price levels generated by (3). Assuming that the total number of critical price levels is finite, we reindex these critical price levels such that $c = p^0 < p^1 < p^2 < \cdots < p^m$, where $m = \sum_i \sum_{j>i} |P_{ij}| + n < \infty$. As we illustrate with some examples later, the sets P_{ij} often contain at most one element, which would lead to $m = O(n^2)$. The preference order of markets is the same within a price interval, $p \in (p^{k-1}, p^k)$. For two consecutive price intervals, (p^{k-1}, p^k) and (p^k, p^{k+1}), the ranking will be the same except that markets i and j will switch places in ordering sequence if $p^k \in P_{ij}$. Hence, we do not need to specifically rank order all markets for each price interval. Instead, we simply rank order them once, and determine which markets switch places at each price breakpoint. For each price interval indexed by $k = 1, \ldots, m$, we solve n maximization problems of the following form:

$$\max_{p \in (p^{k-1}, p^k)} \left\{ \sum_{i=1}^{\ell} R_i(p) - S \sqrt{\sum_{i=1}^{\ell} [C_i(p)]^2} \right\}. \tag{4}$$

After solving (4) for each price interval and for each $\ell = 1, 2, \ldots, n$ within each interval, the optimal solution is characterized by the solution to (4) that results in the highest optimal objective value. Note that, in any given interval, we may discard the markets at the end of the rank ordering with zero demand since they will not be selected anymore.

The running time of the above algorithm depends on the number of threshold price levels, m, and on how fast we can solve the maximization subproblem in the inner loop. Note that if each set P_{ij} contains at most one element and each market has a price where demand becomes zero, there exist $m = O(n^2 + n) = O(n^2)$ price intervals. Hence the running time of the algorithm becomes $O(Tn^3)$ where T denotes the time required to solve (4). In the Appendix, we show that the objective function (4) is concave if $R_i(p)$ is concave and $C_i(p)$ is convex for $p \leq p_i^0$ for all markets. In this case, we can utilize first order conditions to solve the subproblems efficiently.

Under the Newsvendor structure, with both the linear ($q_i(p) = a_i - b_i p$) and iso-elastic ($q_i(p) = \alpha_i p^{-\beta_i}$) demand models, each set P_{ij} contains at most one element when either the coefficient of variation or the standard deviation of demand is constant; that is, either $\sigma_i(p) = cv_i q_i(p)$ or $\sigma_i(p) = \sigma_i$, where cv_i denotes the coefficient of variation. In the linear demand case with a constant coefficient of variation, we have $P_{ij} = \{p_{ij}\}$ if $c \le p_{ij} = (cv_j^2 a_j - cv_i^2 a_i)/(cv_j^2 b_j - cv_i^2 b_i) \le \min(p_i^0, p_j^0)$. Otherwise, $P_{ij} = \emptyset$. With a constant standard deviation, we have $p_{ij} = (\sigma_j^2 a_i - \sigma_i^2 a_j)/(\sigma_j^2 b_i - \sigma_i^2 b_j)$ if $c \le p_{ij} \le \min(p_i^0, p_j^0)$, and $P_{ij} = \emptyset$ otherwise. In the iso-elastic demand case, these price levels are given by $p_{ij} = [(cv_j^2 \alpha_j)/(cv_i^2 \alpha_i)]^{1/(\beta_j - \beta_i)}$ for the constant coefficient of variation case, and $p_{ij} = [(\sigma_j^2 \alpha_i)/(\sigma_i^2 \alpha_j)]^{1/(\beta_i - \beta_j)}$ for the constant standard deviation case. Note that these results again guarantee that $m = O(n^2 + n) = O(n^2)$ price intervals. Hence, the running time of the algorithm becomes $O(Tn^3)$ for all cases discussed above.

Under the EOQ structure, the ratio $R_i(p)/[C_i(p)]^2$ is equal to $(p - c_i)/h_i$ for a given price p, and thus P_{ij} again contains at most one element ($p_{ij} = (c_i h_j - c_j h_i)/(h_j - h_i)$), so that $m = O(n^2)$, and the running time of the algorithm is $O(Tn^3)$. When the procurement cost is the same for all markets (i.e., $c_i = c \; \forall i = 1, \ldots, n$), we have $P_{ij} = \emptyset$ for all (i, j) pairs, and for any price the rank order of the markets is the same, which is determined by the holding cost rates. Hence, the price intervals will be given by only $O(n) p_i^0$ values, and the running time of the algorithm is then $O(Tn^2)$. When the holding cost rate is also the same for all markets, the ratio $R_i(p)/[C_i(p)]^2$ gives the same value at any price level for any market, which indicates that either all or none of the markets with a positive demand rate will be selected. Hence, the price intervals will again be formed only by the p_i^0 values. However, in this case, there is only a single subproblem in each price interval, and the running time of the algorithm is $O(Tn + n \log n)$. The solution procedure described above can also be applied to the case where the cost contribution of each market is constant, i.e., $C_i(p) = C_i > 0$. We next consider the problem in contexts that permit market-specific pricing.

4.2 Market selection with market-specific prices—(MSP-MS)

In this section, we allow different prices in different markets. The notation remains the same except that p, which was the single price in the previous section, is replaced by the price vector $\bar{p}$ with components p_i. Despite the similarity to the single price case, the previous solution approach will not work for this problem since the concept of price intervals no longer exists, which complicates the problem significantly. Note that we assume that the price in a market does not affect revenue (and thus demand) in other markets, an assumption that is appropriate, for example, when customers in a market do not have access to other markets.

For (MSP-MS), we assume that the cost contribution of market i ($C_i(p_i)$) is a nonincreasing function of price and converges to zero when (expected) revenue term ($R_i(p_i)$) is zero, i.e., there exists a price level, p_i^0 such that $R_i(p_i^0) = 0$ and $C_i(p_i^0) = 0$. This assumption makes sense for the following reason: p_i^0 is usually characterized by the (expected) demand function in a market. That is, it is the price level beyond which there is no demand. When there is no demand, it is reasonable to assume that the cost of serving the market is also zero since the concept of serving a market without a demand is not meaningful. Hence, beyond a certain price level, the ideas of serving a market and the associated revenues and costs become irrelevant since there is no demand. We can thus eliminate the market selection variables, since market selection decisions can be inferred from the pricing decisions. In other words, $p_i = p_i^0$ is equivalent to $y_i = 0$, resulting in no revenue or cost associated with that market. (Note that although this assumption also makes sense for the original problem, i.e., market selection with single-price strategy, it does not help with the solution since the firm cannot set different prices in different markets.) Then, the problem reduces to

$$\begin{array}{ll} \max & \sum_{i=1}^{n} R_i(p_i) - S\sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2} \\ \text{subject to} & 0 \le p_i \le p_i^0, \quad \forall i = 1, \ldots, n, \end{array}$$

which is a continuous optimization problem, and the characteristics of the optimal solution are highly dependent on the form of $R_i(p_i)$ and $C_i(p_i)$. If $R_i(p_i)$ is concave and $C_i(p_i)$ is convex for all $i = 1, \ldots, n$, the resulting formulation is a concave maximization problem as shown in the Appendix. This leads to the following proposition.

Proposition 1 *If $R_i(p_i)$ is concave and $C_i(p_i)$ is convex, either all or none of the markets will be selected in the market-specific pricing case.*

Proof Please see Appendix. □

Proposition 1 implies that either the price in each market will equal p_i^0 or will be strictly less than that; that is, either all demands are zero or all markets have strictly positive demand.

When the cost contribution of each market is independent of price (e.g., when the standard deviation of demand in the newsvendor model is independent of price), Proposition 1 does not hold since such a case violates the assumption that $C_i(p_i)$ converges to zero when (expected) revenue term $R_i(p_i)$ is zero, i.e., there exists a price level, p_i^0 such that $R_i(p_i^0) = 0$ and $C_i(p_i^0) = 0$. However, the objective function of this problem is separable by markets and we can solve for the optimal price of each market individually. Let p_i^* denote the optimal price for market i. Since the market selection variable is zero for an unselected market, the prices in such markets can be set arbitrarily and we can reformulate the problem as

$$\begin{array}{ll} \max & \sum_{i=1}^{n} R_i(p_i^*)y_i - S\sqrt{\sum_{i=1}^{n} C_i^2 y_i} \\ \text{subject to} & y_i \in \{0, 1\}, \quad \forall i = 1, \ldots, n. \end{array} \tag{5}$$

This problem has the same form as the selective newsvendor problem (Taaffe et al. [31]).

4.3 Application of the algorithms: a newsvendor example

Consider a newsvendor setting with three markets, $M1$, $M2$, and $M3$. Demand in each market is characterized by its expected value, $q_i(p_i)$, and coefficient of variation, cv_i. We have $q_1(p_1) = 100 - p_1$, $q_2(p_2) = 150 - p_2$, $q_3(p_3) = 200 - p_3$, and $cv_1 = 1/3$, $cv_2 = 1/7$, $cv_3 = 1/7$. Shortage and unit costs, and the salvage value are $e = 300$, $c = 40$, and $v = 20$, respectively. Then, we have $S = K(z^*) = 38.183$; see Sect. 3.1 for details. We next discuss the application of our algorithm for the (MSP-S) case ($p_1 = p_2 = p_3 = p$), and then consider the results for the (MSP-MS) case.

We first generate the critical price levels for each pair of markets in the (MSP-S) case. Note that this example considers a linear expected demand and a constant coefficient of variation for each market. Hence, we know that there is at most one solution to Eq. 3 for each pair of markets. Solving Eq. 3 for each pair, we get $P_{12} = \{88.75\}$, $P_{13} = \{77.5\}$, and $P_{23} = \varnothing$. Including the p_i^0 values, we reindex the critical price levels as follows: $p^0 = c = 40$, $p^1 = 77.5$, $p^2 = 88.75$, $p^3 = 100$, $p^4 = 150$, and $p^5 = 200$. We only rank

Table 1 Summary of the algorithm for the example

					Markets selected					
		Rank order			$R1$		$R1$, $R2$		$R1$, $R2$, $R3$	
k	(p^{k-1}, p^k)	$R1$	$R2$	$R3$	p^*	$G(p^*)$	p^*	$G(p^*)$	p^*	$G(p^*)$
1	(40,77.5)	$M2$	$M3$	$M1$	77.5	2,323	77.5	6,536	77.5	7,329
2	(77.5, 88.75)	$M2$	$M1$	$M3$	88.75	2,652	85.2	3,197	88.75	8,250
3	(88.75,100)	$M1$	$M2$	$M3$	88.75	405	88.75	3,171	96.38	8,431
4	(100,150)	$M2$	$M3$	–	100	2,727	109.30	8,564	–	–
5	(150,200)	$M3$	–	–	150	5,227	–	–	–	–

order the markets at p^0, which corresponds to the interval (p^0, p^1). At subsequent critical price levels, either two markets switch or one of the markets is dropped since the expected revenue becomes zero. Table 1 summarizes the solution of the example.

Within each interval, we solve at most 3 subproblems. To illustrate, let us consider interval 2, i.e., (77.5, 88.75). The rank order of markets is given as $(R1, R2, R3) = (M2, M1, M3)$. The first subproblem in this interval deals with selecting the market that is ranked first, i.e., $M2$. The optimal price and the associated expected profit for this subproblem are 88.75 and 2,652, respectively. The second and third subproblems select $(R1, R2) = (M2, M1)$, and $(R1, R2, R3) = (M2, M1, M3)$, respectively. Having solved all subproblems, the optimal solution for this interval is $p^* = 88.75$ and the corresponding expected profit is 8,250. Note that this is a local optimal solution for the general problem. The global optimal is the largest of the local optimal solutions calculated for all intervals. In this example, it is $p^* = 109.3$, which corresponds to the optimal solution of interval 4. The associated expected profit is 8,564.

An important characteristic of the algorithm for the (MSP-S) problem is that it provides a set of additional (suboptimal) solutions for the entire feasible region in terms of the price variable, and it can easily be modified to capture additional constraints on the price level of the product. For instance, let's assume that the firm does not want to charge more than 90 for this particular example. Then, the optimal solution is found by considering the first three intervals only, where the third interval is modified to be (88.75, 90). Another restriction that can be handled can be explained as follows: assume that the firm wants to serve specific markets. Then, we only need to consider the intervals and associated subproblems that select these markets. For instance, say $M1$ must be served in the above example. In such a case, only subproblem 3 of interval 1, subproblems 2 and 3 of interval 2, and all subproblems of interval 3 should be considered. The associated optimal solution is $p^* = 96.38$.

We now solve the same example for the (MSP-MS) case, allowing different prices in different markets. Recall that we can eliminate the market selection variables, and the resulting formulation is a concave maximization problem since $R_i(p_i) = (p_i - c)(a_i - b_i p_i)$ is concave and $C_i(p_i) = cv_i(a_i - b_i p_i)$ is convex. The optimal solution is provided in Table 2 along with the solution for the (MSP-S) case. In comparison to (MSP-S), (MSP-MS) not only selects the first market in addition to the others, but also generates more profit in markets 1 and 2 due to the flexibility to set different prices. As a result it provides 13.15% higher profits than (MSP-S).

Table 2 Optimal solutions for (MSP-MS) and (MSP-S)

	p_1	p_2	p_3	$R_1(p_1)$	$R_2(p_2)$	$R_3(p_3)$	$S \times C(\bar{p}, y)$	$G(\bar{p}, y)$
(MSP-MS)	73.48	96.29	121.88	887.90	3023.33	6396.46	617.52	9690.16
(MSP-S)	–	109.30	109.30	–	2820.44	6285.56	542.25	8563.75

5 Computational analysis

This section discusses a set of computational tests on a two-market problem with both deterministic and stochastic demand models, which provides insight on market selection decisions and the value of market-specific versus single-price strategies. This analysis considers the impacts that relative market sizes and cost/demand parameters have on pricing and market selection decisions. For stochastic cases in which the model is repeatedly applied (as a sequence of single-period problems), we can gain some insight into how these decisions change as markets expand or contract. More generally, this analysis illustrates the benefits that the flexibility of market-specific pricing provides (relative to a single-price strategy) under a broad range of relative market sizes, price sensitivities and service levels. We focus on the two-market case in order to perform comparative statics to assess how different factors influence the attractiveness of a given market. Although we only consider two markets, one of the markets might correspond to a collection of existing markets, with the other corresponding to an individual market whose value we would like to characterize.

We consider two demand models that are widely used in the literature: linear and iso-elastic demand functions. The linear demand model is represented by a linear price-demand curve for each market $i \in \{1, 2\}$, i.e., $q_i(p) = a_i - b_i p$ where $a_i, b_i > 0$ and $p \leq a_i/b_i$, and the iso-elastic demand model is represented by $q_i(p) = \alpha_i p^{-\beta_i}$ where $\alpha_i > 0$ and $\beta_i > 1$. The parameters a_i and α_i denote the potential market size, and b_i and β_i denote the price sensitivity of demand. Since it possesses the basic model characteristics and enables closed-form solutions, we begin by providing analytical results for the deterministic single-period model. We then provide computational analysis for the newsvendor and EOQ models and conclude that many of the basic observations about the system are the same as for the deterministic model. For these models, we also investigate the effects different parameters have on market selection decisions and the profitability of both models. We distinguish the structural differences between the linear and iso-elastic demand models under these settings.

5.1 Deterministic single-period model

The deterministic single-period model is a special case of the newsvendor model of Sect. 3.1 obtained by setting $\sigma_i(p_i) = 0, \forall i$. We start our analysis with the linear demand model, $q_i(p_i) = a_i - b_i p_i$. In order to analyze the effect of market sizes on market selection decisions, we treat a_1 as fixed and derive threshold values of a_2 that result in different qualitative decisions. Later, we perform the same analysis for price sensitivities, b_i. Recall that we only consider values such that $a_2 - bc > 0$. In the market-specific pricing model, both markets will be selected for all a_2 values since they have positive demands, resulting in a total profit of $\frac{(a_1-bc)^2+(a_2-bc)^2}{4b}$. In the single-price case, there are three possible decisions; either select both markets, select only market 1 (M1), or select only market 2 (M2), resulting in an optimal profit of $\max\left\{\frac{(a_1+a_2-2bc)^2}{8b}, \frac{(a_1-bc)^2}{4b}, \frac{(a_2-bc)^2}{4b}\right\}$. Analyzing this expression, we can

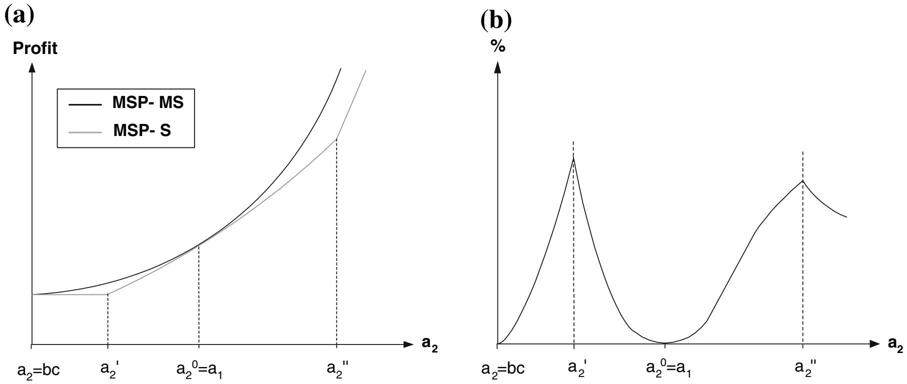

Fig. 1 Deterministic demand analysis: effects of a_2. (**a**) Profits as a function of a_2: (**b**) Percentage difference in profits

obtain threshold values of a_2 (which we denote by a_2' and a_2'') at which the market selection decisions change.

- If $a_2 < a_2' = (\sqrt{2}-1)(a_1 - bc) + bc$, only M1 is selected.
- If $a_2' < a_2 < a_2'' = \frac{a_1 - bc(2-\sqrt{2})}{\sqrt{2}-1}$, both markets are selected.
- If $a_2 > a_2''$, only M2 is selected.

Figure 1 illustrates the profits and the percentage difference in profits between the market-specific pricing (MSP-MS) and single-price (MSP-S) cases. Note that the profit in the (MSP-MS) case is strictly increasing in a_2. For the (MSP-S) case, it is constant up to a_2' since M2 is not selected if $a_2 < a_2'$, and the optimal price is constant on this interval for M1. When market sizes are equal, i.e., at $a_2 = a_1$, the profits are also equal. When $a_2 > a_2''$, only M2 is selected in the (MSP-S) case and the difference in profits is constant thereafter, and equal to the profit generated in M1 in the (MSP-MS) case. This analysis illustrates the value that the flexibility of market-specific pricing provides as market sizes differ. It also illustrates the fact that a threshold value exists for a market's size at which point the market becomes an attractive market to supply under a single-price strategy. More interestingly, if some market (M1 in this case) maintains a constant size and another market (M2) grows, a threshold market size exists for the growing market at which point it becomes attractive to drop the market with a constant size (again, assuming a single-price strategy).

Similar arguments are also valid when we fix the potential market size for both markets to $a_1 = a_2 = a$, set $b_1 = b$ and vary b_2 (see Fig. 2). In this case, for the single price case, there exist threshold values b_2' and b_2'' such that only M2 is selected when $b_2 \leq b_2'$, both markets are selected when $b_2' < b_2 \leq b_2''$, and only M1 is selected when $b_2 > b_2''$.

When the demand is iso-elastic, both markets will be selected regardless of the pricing scheme as both will have positive demands at any price level. Moreover, the optimal price of a single market is given by $\beta c/(\beta - 1)$, which is independent of the potential market size. Hence, when both markets have the same price sensitivity, $\beta_1 = \beta_2 = \beta$, the optimal price is given by $p = \beta c/(\beta - 1)$, and market-specific pricing and single pricing strategies coincide, even with different potential market sizes.

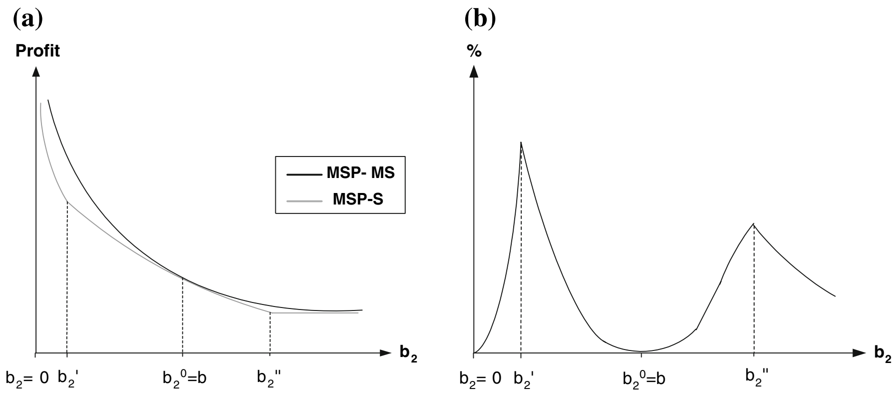

Fig. 2 Deterministic demand analysis: effects of b_2 (**a**) Profits as a function of b_2: (**b**) Percentage difference in profits

5.2 Newsvendor model

In this section, we first highlight the similarities between the deterministic and the stochastic single-period models. Since we are unable to provide closed-form analytical results, we generate a test case that strongly resembles the analysis of the deterministic case. Let $q_1(p) = 500 - 3p$, $q_2(p) = a_2 - 3p$ and $\sigma_i(p) = q_i(p)/3$; also let $(c, v, e) = (100, 50, 300)$. The expected profits and the percentage difference in expected profits as functions of a_2 are similar to those for the deterministic case. The smallest size at which M2 is selected is 412.72 for this example. The corresponding value in the deterministic case for the same parameters is 382.84, which is intuitively reasonable since uncertainty results in higher costs, and a greater expected demand is required to enter a market with uncertainty. Likewise, the threshold value of a_2 at which M1 is no longer selected is 711.15 in the stochastic case, whereas it is 782.84 when demand is deterministic. Note also that the highest percentage differences in profit also occur at these threshold values for both models.

We next perform a thorough computational analysis to analyze the market selection and pricing decisions in a newsvendor setting, and to highlight the differences between the 'single pricing' and 'market-specific pricing' strategies.

5.2.1 Linear demand model

We start with the linear demand model and consider all combinations of the following parameter sets: $(a_1, a_2) = \{(450, 450), (650, 450), (850, 450), (650, 650), (850, 650), (850, 850)\}$, $b_1 = \{2, 3, 4\}$, $b_2 = \{2, 3, 4\}$, $e = \{200, 300, 400\}$, $c = 100$, $v = 50$, $cv_1 = \{1/7, 1/5, 1/3\}$ and $cv_2 = \{1/7, 1/5, 1/3\}$, where cv_i denotes the coefficient of variation in market i. Note that although our models and solution approaches introduced in Sects. 3 and 4 apply under general standard deviation functions, we assume $\sigma_i(p) = q_i(p)/cv_i$ to employ a single parameter for the uncertainty in the system, since our primary goal is to analyze the effects of such parameters on market selection decisions. The appendix contains a discussion of the implications of using different functional forms for the representation of $q_i(p)$ and $\sigma_i(p)$.

Table 3 provides the market selection decisions for (MSP-S) under different market sizes and price sensitivities where the entries are of the form (y_1^*/y_2^*). For all cases except when $a_1 = a_2 = 450$ and $b_1 = b_2 = 4$, the selection decisions are consistent across all remaining

Table 3 (MSP-S) Market selection decisions for different demand parameter values

	$a_1 - a_2$					
$b_1 - b_2$	450 − 450	650 − 450	850 − 450	650 − 650	850 − 650	850 − 850
2 − 2	1/1	1/1	1/0	1/1	1/1	1/1
2 − 3	1/0	1/0	1/0	1/1	1/0	1/1
2 − 4	1/0	1/0	1/0	1/0	1/0	1/0
3 − 2	0/1	1/1	1/1	1/1	1/1	1/1
3 − 3	1/1	1/0	1/0	1/1	1/1	1/1
3 − 4	1/0	1/0	1/0	1/1	1/0	1/1
4 − 2	0/1	1/1	1/1	0/1	1/1	0/1
4 − 3	0/1	1/1	1/0	1/1	1/1	1/1
4 − 4	*/*	1/0	1/0	1/1	1/1	1/1

parameter settings, whereas for this particular case, the selection decisions are not consistent across all other parameter settings. In this case, however, the expected profit of the firm is so low that the market selection decisions are relatively unimportant. Hence, we do not consider this case for further analysis.

Observation 1 *Under (MSP-S), a market that is not selected at a low market size may become attractive when potential market size increases or price sensitivity decreases. More interestingly, increasing (decreasing) potential market size (price sensitivity) might cause the other market, which is attractive currently, to be dropped.*

Consider the case where $(a_1, a_2) = (450, 450)$ and $(b_1, b_2) = (3, 2)$, for instance. In this case, only M2 is selected. As b_2 increases, M2 becomes less attractive and is dropped when $b_2 = 4$. Although there is no corresponding change in its parameters, M1 becomes more attractive and is selected when $b_2 = 3$ or $b_2 = 4$. When the sensitivity of M1 is high relative to that of M2, M1 is not selected; however, when the same sensitivity is relatively small, it becomes the only profitable option.

Observation 2 *Under (MSP-S), an increase in the coefficient of variation of a selected market's demand results in an increase in the price for all markets.*

In this case, the firm's only tool to compensate for increased uncertainty in any market is the single price, and both markets are negatively affected by the resulting increase in price. Recall that one of the favorable aspects of single pricing is the fairness among markets that it provides. However, Observation 2 indicates that under (MSP-S), customers in a market are negatively affected by a change in another market, which calls into question the actual fairness of a single-price strategy.

Observation 3 *A market that is currently selected may be dropped because of an increase in the coefficient of variation of the demand in any one of the markets.*

Note that the increasing price that results from a market's increase in uncertainty (see Observation 2) implies that the expected demands in both markets decrease. This decrease may cause a smaller market to be dropped because this provides the firm an opportunity to further

Table 4 (MSP-S) Market selection decisions for the example

cv_1	cv_2	p	y_1	y_2	Profit	$(a_1 - b_1 p)y_1$	$(a_2 - b_2 p)y_2$
1/3	1/3	154.85	0	1	2185.27	0.00	69.34
1/3	1/5	150.18	0	1	2880.22	0.00	79.60
1/3	1/7	133.58	1	1	3283.08	18.54	116.12
1/5	1/3	154.85	0	1	2185.27	0.00	69.34
1/5	1/5	150.18	0	1	2880.22	0.00	79.60
1/5	1/7	132.30	1	1	3342.29	22.65	118.95
1/7	1/3	154.85	0	1	2185.27	0.00	69.34
1/7	1/5	132.48	1	1	2892.96	22.08	118.55
1/7	1/7	131.84	1	1	3363.65	24.11	119.95

increase price in larger markets and possibly generate greater profit. The parameter set given at the beginning of this section does not provide an example for the case where a market is dropped due to an increase in the coefficient of variation. In order to analyze this phenomenon more closely, we consider the following example.

Example Let $(a_1, a_2) = (446, 410)$, $(b_1, b_2) = (3.2, 2.2)$, $e = 300$, $c = 100$ and $v = 50$. The selection and pricing decisions of (MSP-S) for this example are reported in Table 4. Note that the first market is dropped when the coefficient of variation of either market increases. Interestingly, M1 is more vulnerable to changes in cv_2 than it is to cv_1. For instance, when $cv_2 = 1/7$, M1 is not dropped even when cv_1 is as large as $1/3$. On the other hand, even if $cv_1 = 1/7$, it is dropped when $cv_2 = 1/3$, which can be explained as follows. Since the expected demand in M1 is far less than M2, an increase in cv_1 does not have a substantial effect on the aggregate standard deviation seen by the supplier. Hence, the supplier can still afford to select M1. However, an increase in cv_2 would result in considerably larger aggregate standard deviation. In this case, the firm may drop M1 to further increase the price and balance the increase in standard deviation. □

Another parameter that affects selection and pricing decisions is the shortage cost. As shortage cost increases, the cost of the firm due to uncertainty increases. Hence, we expect that the firm would increase the price to decrease the aggregate variation. Similar to the results for an increase in coefficient of variation, increasing shortage cost also affects the selection decisions under (MSP-S); that is, an increase in shortage cost may force the firm to drop a currently selected market under (MSP-S). Consider the example above with $cv_1 = 1/7$ and $cv_2 = 1/5$. Both markets are selected when the shortage cost is \$300. When it increases to \$400, the supplier increases the price as an attempt to decrease the uncertainty, which causes the first market to be dropped.

We now compare the single-price strategy to the market-specific pricing strategy. Table 5 reports the average percentage difference in profits for different market sizes and price sensitivities. When attempting to interpret trends in these percentage differences, we must keep in mind that the corresponding market selection decisions may change as we change parameter values (see Table 3). We can, however, draw certain conclusions based on these results, the first of which is fairly intuitive.

Table 5 (MSP-MS) vs. (MSP-S)—average percentage differences in profits

	$a_1 - a_2$					
$b_1 - b_2$	450 – 450	650 – 450	650 – 650	850 – 450	850 – 650	850 – 850
2–2	0.19	11.12	0.06	15.42	4.01	0.03
2–3	25.67	7.59	14.37	3.60	19.71	9.87
2–4	1.99	0.61	15.40	0.29	7.33	23.99
3–2	25.67	0.26	14.37	4.98	1.20	9.87
3–3	1.42	20.88	0.22	8.01	7.10	0.09
3–4	10.66	1.65	17.24	0.65	16.29	8.20
4–2	1.99	19.78	15.40	0.31	15.80	23.99
4–3	10.66	4.90	17.24	17.48	0.23	8.20
4–4	20.62	5.14	0.86	1.40	15.57	0.24

Observation 4 *The advantage of offering different prices in different markets becomes quite small when (a) the prices in the (MSP-MS) case are close to each other; (b) one of the markets generates much less profit than the other.*

In the first case (part (a)), (MSP-S) selects both markets and the optimal single price is close to the optimal market-specific prices for both markets. In the second case (part (b)), (MSP-S) selects the market with the higher profit only. Since the other market does not contribute to the profits significantly, it does not hurt (MSP-S) not to select it.

Observation 5 *Under (MSP-MS), an increase in the coefficient of variation of a market's (say M1) demand causes the price in that market to increase whereas the price in the other market (say M2) decreases.*

The first part of Observation 5 is obvious; since the standard deviation increases with the market demand, the firm increases the price to balance the increase in uncertainty. The fact that a market's price may decrease in response to an increase in another market's uncertainty, however, is surprising, and can be explained as follows. Because M2 becomes relatively less uncertain, the supplier decreases the price in this market to balance the decrease in total demand due to the higher price required in M1. Hence, we may conclude that the buyers in M2 face a lower price as a result of an increase in uncertainty in M1, although the characteristics of M2 are unchanged.

In summary, with a linear demand model, our computational results indicate that potential market size and price sensitivity are critical factors in driving market selection decisions, although coefficient of variation and shortage costs may also play a significant role in certain situations. (MSP-MS) always outperforms (MSP-S) as expected. Yet, the magnitude of the difference depends on the relative cost parameters, the similarities between markets in terms of resulting pricing decisions, and the coefficient of variation of the demands. We also observe that a market may be negatively affected (even dropped) because of the changes in the other market under (MSP-S), which makes the fairness assertion of (MSP-S) questionable.

5.2.2 Iso-elastic demand model

We next consider the iso-elastic demand model and highlight its differences from the linear model. The expected demand in market i is given by $d_i(p_i) = \alpha_i p_i^{-\beta_i}$, and we consider all the combinations of the following parameter sets (where 1M = 1 million):
(α_1, α_2) = {(1M, 1M), (2M, 1M), (2M, 2M), (3M, 1M), (3M, 2M), (3M, 3M)}, $\beta_1 = \{1.4, 1.7, 2\}, \beta_2 = \{1.4, 1.7, 2\}, e = \{200, 300, 400\}, c = 100, v = 50, cv_1 = \{1/7, 1/5, 1/3\}$ and $cv_2 = \{1/7, 1/5, 1/3\}$.

Observation 6 *With the iso-elastic demand, both markets are selected in all instances regardless of the pricing strategy.*

This results because the iso-elastic demand model implies positive demand for any price level in each market. Hence, contrary to the linear demand model, when it is optimal for the supplier to set a relatively high price, all markets are still technically selected, although this may imply very small demand levels in markets that would have otherwise been dropped under the linear demand model.

Observation 7 *As in the linear demand model, under (MSP-S), an increase in the uncertainty of a market's demand results in an increase in the price.*

Observation 8 *The percentage difference of expected profits under the (MSP-MS) and (MSP-S) strategies are not as large as in the linear demand model.*

Observation 9 *As in the linear demand model, under (MSP-MS), an increase in the uncertainty of a market's demand causes the price in that market to increase whereas the price in the other market decreases.*

In summary, with an iso-elastic demand model, both markets are effectively "selected" under both pricing schemes. The difference in profits generated by different pricing schemes is not as significant as in the linear demand model. Hence, the supplier may employ the single pricing scheme without significant loss if the demands in the markets are iso-elastic. The pricing decisions are affected by a change in the cost/uncertainty parameters in the same way as the linear model.

Throughout our analysis of the newsvendor model, we have assumed a constant coefficient of variation for both the linear and iso-elastic demand models, in order to understand the effects of uncertainty on market selection and pricing decisions. In order to examine the validity of our observations in the absence of this assumption, we performed computational tests with more general $\sigma_i(p)$ functions to see whether our findings apply to those cases. As we discuss in greater detail in the Appendix, under mild assumptions (in particular, if $q_i(p)$ and $\sigma_i(p)$ approach zero at the same point or at the same rate) the observations we have discussed continue to hold.

5.3 Economic order quantity model

We performed an extensive computational study on the EOQ model by considering different values of market sizes, demand sensitivities, holding and fixed ordering costs. Our analysis reveals that the market selection decisions are almost identical to the newsvendor model, which is quite intuitive since both models can be represented with almost the same mathematical model. Our observations in Sect. 5.2 regarding the relation between market selection and potential market size and price sensitivity, and the difference in expected profits are also valid for the EOQ model. Moreover, the effects of holding and fixed ordering costs on market selection and pricing decisions are similar to the effects of uncertainty and shortage cost in the newsvendor model, respectively. Hence, we do not provide further details on the computational results regarding the EOQ model.

6 Conclusion and research directions

This study considered a nonlinear optimization model that involves binary variables together with continuous variables, and introduced an efficient solution algorithm for this problem. The formulation applies to a number of inventory problems involving simultaneous market/order selection, pricing and quantity decisions under certain assumptions. Two inventory models were explicitly discussed. The first one is a newsvendor-type problem in multiple markets with pricing, where a supplier chooses the optimal set of markets to supply. We incorporated randomness in each market's demand in such a way that the standard deviation of the demand increases with the expected demand and showed that both multiplicative and additive randomness models can be handled in this manner. The second one is an EOQ type problem in multiple markets with pricing. We investigated two different pricing strategies for these models, where (i) the firm chooses to offer a single price in all selected markets, (MSP-S) and (ii) for each market, a different price is set, (MSP-MS).

The solution algorithm for the 'single-price' strategy employs the 'Decreasing Expected Revenue to Uncertainty' (DERU) ratio (Taaffe et al. [31]) to determine relative market attractiveness for any given price. In particular, we generate a finite number of price intervals within which the rank order of the markets according to the DERU ratio does not change, and for each interval, we find the optimal price. This idea then leads to an efficient algorithm that solves the market selection problem with a single-price constraint. Under the 'market-specific pricing' strategy, we first observe that the market selection variables can be omitted. This yields a continuous nonlinear objective function in the price variables. Under mild conditions, we showed that the objective function is jointly concave, and hence the problem is efficiently solvable. We performed an extensive numerical analysis in order to further understand the dynamics of both strategies and observe their reactions to changing market conditions and cost parameters.

In practical contexts, the exact demand curve and cost parameter values may not be known with certainty and will thus require estimation. While the approach we have outlined for the newsvendor version of the problem accounts for demand uncertainty to a certain degree, we recognize the limitations of the assumption of deterministic cost parameters and demand curves in practice. Thus, implementation of our models in practice should be accompanied by a thorough sensitivity analysis, in order to understand how parameter variations affect the prescribed solution. An immediate extension to this work would consider a multi-period problem where the demands are nonstationary. In this setting, the firm may change the selection decision from one period to another subject to incurring a certain cost. Another assumption of the formulation is that the pricing decision in one market does not affect the demand in other markets. Although this may be a reasonable assumption for geographically dispersed markets, there are also cases where customers may travel to other markets with lower prices, which would lead to another extension with a form of demand substitution.

Appendix

Equivalence of additive and multiplicative randomness

Let the demand in each market be $D_i = q_i(p)X_i$ where X_i is normally distributed with mean 1 and standard deviation σ_i. Let X_y denote the aggregate demand, i.e., $X_y = \sum_{i=1}^n D_i y_i$. Note that X_y is also normally distributed with mean $\sum_{i=1}^n q_i(p)y_i$ and standard deviation $\sqrt{\sum_{i=1}^n [q_i(p)]^2\sigma_i^2}$. Denoting the pdf of X_y by $f_y(x, p)$, the expected profit of the firm is

$$G(p, Q, y) = (p - c)\sum_{i=1}^{n} q_i(p)y_i - (c - v)\int_{-\infty}^{Q} (Q - x)f_y(x, p)dx - (e - c)\int_{Q}^{\infty}(x - Q)f_y(x, p)dx.$$

Optimizing over Q given y and p and substituting optimal order quantity in the expected profit, the problem reduces to

$$\begin{array}{lll} \max & (p - c)\displaystyle\sum_{i=1}^{n} q_i(p)y_i - K\sqrt{\sum_{i=1}^{n}[q_i(p)]^2\sigma_i^2 y_i} & (6) \\ \text{subject to} & y_i \in \{0, 1\} \quad \forall i = 1, \ldots, n. & \end{array}$$

The only difference between the multiplicative and additive randomness is the standard deviation of the random variable X_y. If we let the standard deviation of additive randomness in each market, $\sigma_i(p) = q_i(p)\sigma_i$, where σ_i is the standard deviation of X_i in the multiplicative case, then both models have the same structure. Hence, we can conclude that multiplicative randomness is a special case of the additive model and can be handled in exactly the same way. Similar arguments are also valid when the firm is able to offer market-specific prices.

Proof of Concavity: (MSP-S)

The objective function of each subproblem is given by $G(p) = \sum_{i=1}^{\ell} R_i(p) - S\sqrt{\sum_{i=1}^{\ell}[C_i(p)]^2}$.

Since each $R_i(p)$ is assumed to be concave, the first part of $G(p)$ is concave. Below, we show that $C(p) = \sqrt{\sum_{i=1}^{\ell}[C_i(p)]^2}$ is convex. Hence, we can conclude that $G(p)$ is concave.

The second derivative of $C(p)$ with respect to p is given by

$$\frac{d^2C(p)}{dp^2} = \left[\sum_{i=1}^{\ell}[C_i(p)]^2\right]^{-\frac{3}{2}} \left[-\left(\sum_{i=1}^{\ell} C_i(p)C_i'(p)\right)^2 + \left(\sum_{i=1}^{\ell}[C_i(p)]^2\right) \left(\sum_{i=1}^{\ell}([C_i'(p)]^2 + C_i(p)C_i''(p))\right)\right]$$

where $C_i'(p)$ and $C_i''(p)$ denote the first and second derivatives of $C_i(p)$, respectively. Let $T = \left[\sum_{i=1}^{\ell}[C_i(p)]^2\right]^{-\frac{3}{2}}$. Also denote $C_i(p)$, $C_i'(p)$, and $C_i''(p)$ by A_i, B_i and C_i. Then,

$$\begin{aligned}
\frac{d^2C(p)}{dp^2} &= T\left[\left(\sum_{i=1}^{\ell} A_i^2\right)\left(\sum_{i=1}^{\ell}\left[B_i^2 + A_iC_i\right]\right) - \left(\sum_{i=1}^{\ell} A_iB_i\right)^2\right] \\
&> T\left[\left(\sum_{i=1}^{\ell} A_i^2\right)\left(\sum_{i=1}^{\ell} B_i^2\right) - \left(\sum_{i=1}^{\ell} A_iB_i\right)^2\right] \\
&= T\left[\sum_{i=1}^{\ell}\sum_{j\neq i} A_i^2B_j^2 - 2\sum_{i=1}^{\ell}\sum_{j>i} A_iB_iA_jB_j\right] \\
&= T\left[\sum_{i=1}^{\ell}\sum_{j>i}(A_iB_j - A_jB_i)^2\right] > 0
\end{aligned}$$

□

Proof of Concavity: (MSP-MS)

The objective function of the problem is $\sum_{i=1}^{n} R_i(p_i) - S\sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2}$. The first part is concave since each $R_i(p_i)$ is assumed to be concave. We need to show that $C(p) = \sqrt{\sum_{i=1}^{n}[C_i(p_i)]^2}$ is convex in p. Let $\overrightarrow{C}(p) = [C_1(p_1), \ldots, C_n(p_n)]^T$. Then $C(p)$ is the ℓ_2 norm of $\overrightarrow{C}(p)$, i.e., $C(p) = ||\overrightarrow{C}(p)||$. For any $\lambda \in (0, 1)$, the following holds for any p_1 and p_2:

$$\begin{aligned}
C(\lambda p_1 + (1-\lambda)p_2) = ||\overrightarrow{C}(\lambda p_1 + (1-\lambda)p_2)|| &\leq ||\lambda\overrightarrow{C}(p_1) + (1-\lambda)\overrightarrow{C}(p_2)|| \\
&\leq ||\lambda\overrightarrow{C}(p_1)|| + ||(1-\lambda)\overrightarrow{C}(p_2)|| = \lambda C(p_1) + (1-\lambda)C(p_2)
\end{aligned}$$

The first inequality holds since $C_i(p_i)$'s are all convex. The result directly follows. □

Proof of Proposition 1

We show that if any market is selected in the optimal solution, then market i should also be selected, and if market j is not selected in the optimal solution, then market i will not be selected either for any i.

Since the expected profit function $G(p)$ is jointly concave in p, we can utilize first-order conditions to characterize the optimal solution. The first derivative with respect to p_i is given by

$$\frac{\partial G(p)}{\partial p_i} = R_i'(p_i) - S\frac{C_i(p_i)C_i'(p_i)}{\sqrt{\sum_{j=1}^{n}[C_i(p_i)]^2}} \tag{7}$$

Let p_i^0 denote the minimum price at which $R_i(p_i)$ and hence $C_i(p_i)$ are equal to zero. At p_i^0, $\left(\sum_{j=1}^{n}[C_j(p_j)]^2\right)^{-1/2}$ is well defined since at least one market is selected and the first derivative evaluated at p_i^0 is

$$\left.\frac{\partial G(p)}{\partial p_i}\right|_{p_i=p_i^0} = R_i'(p_i^0) < 0 \tag{8}$$

indicating that the optimal p_i should be less than p_i^0, i.e., market i should be selected.

Assume market j is not selected in the optimal solution. We next characterize the decision regarding market i. If market i is to be selected, then due to the first part of the proof, market j should also be selected, which violates the assumption that market j is not selected. Hence, the optimal decision for market i should be 'not selecting'. □

Different standard deviation functions for the newsvendor model

Recall that in Sect. 5, for the newsvendor model, we only considered $\sigma_i(p) = cv_i \times q_i(p)$ and varied the value of cv_i to examine the effects of uncertainty. We now extend our computations to include general $\sigma_i(p)$ functions. Figure 3 illustrates the nonlinear functions used in the linear and iso-elastic demand cases.

In both cases, the $\sigma(p)$ functions are generated such that the coefficient of variation is less than 1/3 (see Fig. 3). In the linear case, the standard deviation function is given by $\sigma(p) = s - \sqrt{R^2 - (p-t)^2}$. The parameters s, t and R are generated so that $\sigma(a/b) = 0$, and $q(p)/3 > \sigma(p) > 0$ for all $p < a/b$. They also define the level of uncertainty in the market. In the iso-elastic demand case, the standard deviation function is given by $\sigma(p) = \bar{\alpha} p^{-\bar{\beta}}$. The parameters $\bar{\alpha}$ and $\bar{\beta}$ are computed so that $q(p)/3 > \sigma(p) > 0$. For both cases, we generate $\sigma(p)$ functions as in Fig. 3 for each set of demand parameters considered in Sect. 5. Replicating our computational study with these standard deviation functions, we observed that all of our observations are still valid for both the linear and the iso-elastic demand cases.

For the case of linear demand we also consider a linear standard deviation function. Note that the constant coefficient case corresponds to the linear standard deviation function as depicted in Fig. 4a. We now consider a linear function that approaches zero before the demand does. In such a case, the demand becomes deterministic when price exceeds a certain threshold. We generate different levels of uncertainty by changing this threshold price level (See Fig. 4b).

Replicating our computational analysis with this form of the standard deviation function, we found that our observations regarding the market selection decisions still hold. However,

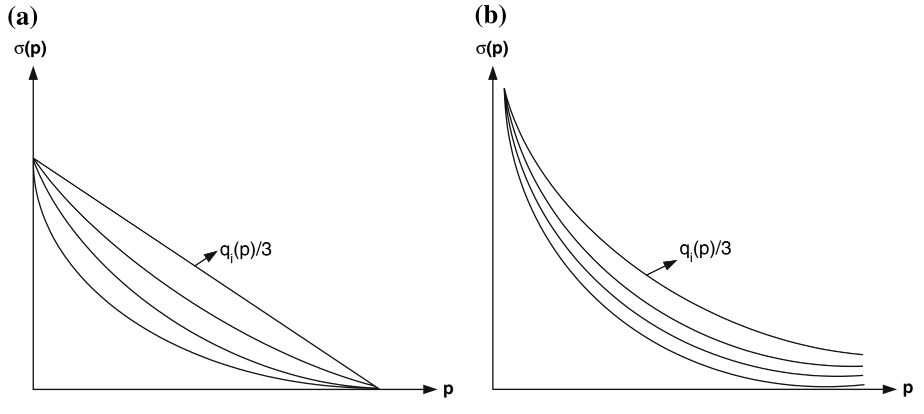

Fig. 3 Standard Deviation. (**a**) $\sigma(p)$ for linear demand. (**b**) $\sigma(p)$ for iso-elastic demand

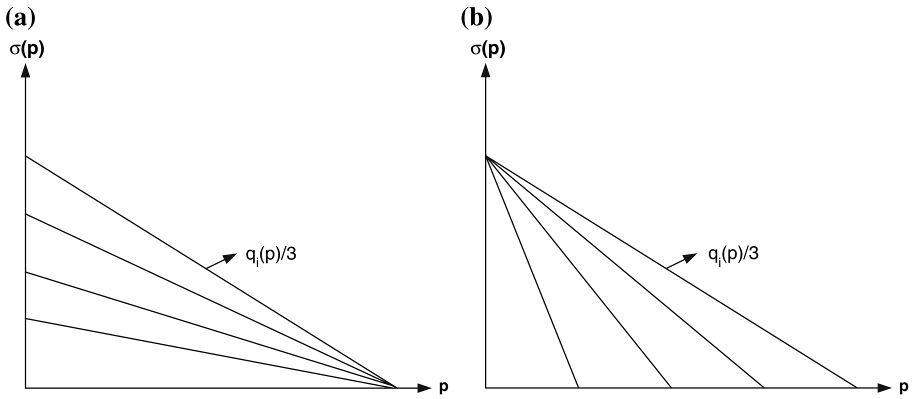

Fig. 4 Linear standard deviation for linear demand. (**a**) Linear $\sigma(p)$ for linear demand: I. (**b**) Linear $\sigma(p)$ for linear demand: II

our observations about the effects of uncertainty on the pricing decisions do not generalize to this case. This is because we have a standard deviation function that allows deterministic demand when price exceeds a certain threshold value. In some instances, the supplier sets the price such that the resulting demand in the market(s) is deterministic. When uncertainty increases, that is, when the threshold price level increases, the supplier may also increase the price to remain in the (smaller) region where demand is effectively deterministic. Further increasing uncertainty increases the threshold price at which demand is effectively deterministic, and may force the supplier to decrease the price in order to increase demand at the expense of incurring some degree of uncertainty. Hence, the reaction of optimal prices to changes in uncertainty is not necessarily consistent in this special case.

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